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FULL-PROFILE CONJOINT ANALYSIS: SOME MEASURING, MODELING AND LEVELS OF AGGREGATION

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Abstract*. In this work the various conjoint analysis (COA) models developed by the author are reviewed. Such models consider different levels of data response measurement scales; different levels of response aggregation, either individual or aggregated and different parameter estimation methods. We therefore report: i) some approaches to full-profile COA by multiple linear regression analysis: weighted least squares approach; the arcosin transformation approach; an additive binary coding of ordinal experimental factors; COA to estimate more than one response function; ii) some approaches to full-profile COA by multiple logistic regression analysis (ordinal logistic regression for the estimate of the response functions; multivariate logistic regression; multivariate logistic regression response with main and interaction effects).*

Keywords*: Conjoint analysis, Multivariate linear regression analysis, Multivariate logistic regression analysis.*

1. INTRODUCTION

Conjoint Analysis (Coa) is a popular marketing research technique. It is used in designing new products, changing or repositioning existing products, redesigning processes. It is used to study the factors that influence consumers' product preferences and simulate consumer choice.

Coa deals with preference data (ratings or rankings) expressed by individuals (consumers, potential buyers, service users, judges, etc.) - in a consumer research - on a set of stimuli (products) described by attributes assuming different values or categories (attribute-levels). Each stimulus is a combination of attribute levels.

Aim of the Coa is to evaluate the relative importance of levels/attributes using only the global preference (*overall*) - known - on the product: the preference model can be multiplicative, additive and decompositive.

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The estimation method attempts to find a set of individual part-worth utilities (by a multiple regression model) that relate the attribute levels of an object to overall evaluation.

Partial utility coefficients can be used for computing the total utility of the stimulus, as well as for estimating the utility of stimuli absent in the experiment.

The stimulus with the highest total utility is the optimum marketing-mix.

A) Some approaches to full-profile conjoint analysis by multiple linear regression analysis

2. CONJOINT ANALYSIS WITH A DICHOTOMOUS DEPENDENT VARIABLE: *WEIGHTED LEAST SQUARES* APPROACH

If Y is the dichotomous dependent variable (*overall response* on a product profile) assuming two judgment categories: $Y_1 = 1$ «satisfactory», $Y_2 = 0$ «unsatisfactory», and $X_1, \ldots X_l, \ldots, X_M$ are the experimental factors, with levels $l = 1, 2, .., l_m$ and $m = 1, 2, \ldots, M$ attributes - expressed with disjunctive binary coding z - the following regression equation, without intercept, is to be considered, which establishes a linear relation between variable Y and independent variables X_m:

$$\mathbf{Y} = \mathbf{Z}\ \boldsymbol{\delta} + \mathbf{E} \tag{1}$$

where:

$\mathbf{Y} = [Y_1, \ldots, Y_K]'$ is the preference column vector of the generic judge, relevant to the K stimuli Y_k, $k = 1, 2, .., K$; Y_k takes values 0 or 1 ($Y \in \{0,1\}$);

$\mathbf{Z}$ = the experimental design matrix of dimensions $K \times \sum_{m=1}^{M} l_m$ with elements z_{ml}, dummy variable relevant to *l-th* level of the generic "*m*" attribute ($m = 1, 2, \ldots, M$);

$\boldsymbol{\delta} = [\delta_1^{(1)}, \ldots, \delta_{1l_1}^{(1)}, \ldots \delta_{l_M}^{(M)}]'$ column vector containing the unknown parameters (*utility coefficients*) associated with the dummy variables of l level of each m attribute (where $\delta_1^{(1)}$ is the constant term associated to the reference category);

$\mathbf{E} = [\ E_1, \ldots, E_K]'$ is the column vector with generic observation E_k indicates the observation error relevant to the k-th stimulus; E_k is heteroskedastic.

Being the dichotomous (0, 1) dependent variable (and considering the heteroskedasticity of the disturbance term), we used the *weighted least squares method* to estimate the model parameters (linear probability model) at the *aggregated* level. Independent variables are also dichotomous.

Indicating with:

$$y_k = \sum_{j=1}^{n_k} y_j \quad (\text{for } Y_j = 1), \tag{2}$$

the fraction of positive responses on the k-th stimulus ($k = 1, 2, \ldots, K$) provided by n judges is: $f_k = \frac{y_k}{n}$.

We can write the regression equation in the following way:

$$f_k = \sum_{m=1}^{M} \sum_{l=2}^{l_m} \delta_l^{(m)} \tilde{z}_{mlk} + \varepsilon_k \tag{3}$$

$\tilde{z}_{mlk}$ indicates a row vector in which the first column has been suppressed, for each of blocks of dummy variables (*baseline*), and it has been included in the column of the constant term.

Considering the heteroskedasticity of the disturbance term we applying to (3) the following system of weights w_k, obtained by the same relative frequencies f_k: $w_k = \left[\frac{n_k}{[(f_k)(1-f_k)]}\right]^{1/2}$; n_k = number of observations (i.e. 100 in the example).

By the system of weights w_k we have the following relationship:

$$w_k \cdot f_k = \sum_{m=1}^{M} \sum_{l=2}^{l_m} \delta_l^{(m)} \cdot w_k \cdot \tilde{z}_{mlk} + w_k \cdot \varepsilon_k \tag{4}$$

By applying the ordinary least squares method (OLS) to the observations thus transformed, we come to the estimate model parameters.

For $np(1-p) > 10$ (p = proportion of the overall response 1 on the combination k), the binomial distribution of p variable approximates the normal variable. If the relation $np(1 - p) > 10$ is verified for all f_k values, we can be use – as regard the estimators obtained by the weighted least squares method – the standard tests of significance of the linear regression.

2.1 APPLICATION

The model was applied to the results of a survey conducted on a sample of college students, who were administered a questionnaire structured according to a sequence of hypothetical profiles of a university course.

The overall evaluation Y toward the service provided by a university was expressed by $n = 100$ students on a dichotomous scale: 1 = *satisfactory*, 0 = *unsatisfactory*, about 27 combinations of levels l judgments (l = 1, 2, 3): 1 = *unsatisfactory*, 2 = *satisfactory*, 3 = *more than satisfactory*, expressed by disjunctive binary coding, with reference to attribute: Teaching, Structures, Services. It is a full factorial design with uniform replications.

The estimated aggregate model:

$$\hat{f} = \hat{p} = 0{,}0009 + 0{,}603\,\tilde{z}_{12} + 0{,}701\,\tilde{z}_{13} + 0{,}138\tilde{z}_{22} + 0{,}195\tilde{z}_{23} + \\ + 0{,}116\tilde{z}_{32} + 0{,}188\tilde{z}_{33}.$$

The linear probability function formulation allows to fall outside the interval between 0 and 1 [0, 1], which is inconsistent with definition of p and with the interpretation of the expectation as a probability.

With reference to this last constraint, if not spontaneously satisfied by sample estimates, in order to comply it (instead of adopting the *arcsine* transformation on the aggregated observations; v. § 3, De Luca, 2004; pp. 349-358), we can truncate the p distribution in the following way: with p_{inf} indicating the lowest value that p can take and p_{sup} its highest value, we assume: $p_{inf} = 0$ e $p_{sup} = 1$.

In the application we have only three values out of 27 for which $\hat{p}_k$ exceeds 1; we have, therefore, the following probability vector:

$$\hat{\mathbf{p}}' = [0{,}0009\ 0{,}12\ \ 0{,}19\ 0{,}14\ 0{,}25\ 0{,}33\ 0{,}20\ 0{,}31\ 0{,}38\ 0{,}60\ 0{,}72\ 0{,}79\ 0{,}74 \\ 0{,}86\ 0{,}93\ 0{,}80\ 0{,}91\ 0{,}99\ 0{,}70\ 0{,}82,\ 0{,}89\ 0{,}84\ 0{,}96\ 1{,}00\ 0{,}90\ 1{,}00\ 1{,}00].$$

3. CONJOINT ANALYSIS WITH DICHOTOMOUS AND LIMITED DEPENDENT VARIABLE: THE *ARCOSIN* TRANSFORMATION APPROACH

Being the dependent variable dichotomous (0, 1), and considering the heteroskedasticity of the disturbance term, the *generalized* least-squares regression is appropriate to estimate the Coa model.

But a distinct difficulty remains: the linear probability function formulation allows to fall off the interval between 0 and 1 [0, 1]. To solve this problem we used the *arcsine transformation* method, to estimate the model parameters, at the aggregated level.

Considering the example discussed discussed in Section 2.1, we preliminarily submit the values p_k to the *arcsine* transformation in order to stabilize their variance.

Indicating with p_k the proportion of respondents that on the combination k, $k = 1, 2, \ldots 27$, gave *overall* response 1 (satisfactory), we have:

$$M(p_k) = P_k\,; \qquad Var\,(p_k) = P_k\,(1 - P_k)/n$$

($n = 100$; P_k = proportion of successes (1) related to the population).

Indicating with φ the angle in radians, obtained by the following transformation: $\varphi = 2arcsin\sqrt{p_k}$ (with: $0 \leq p_k \leq 1$; $0 \leq \varphi \leq \pi$), we have the following inverse function: $p_k = sin^2\frac{\varphi}{2}$; therefore, for n_k sufficiently large, is: $\varphi = M(\varphi) \cong \varphi(P_k)$, with $\varphi(P_k) = 2arsin\sqrt{P_k}$.

The model expression is the following:

$$\mathbf{\Phi}(f) = \tilde{\mathbf{Z}}\tilde{\boldsymbol{\delta}} + \boldsymbol{\varepsilon} \tag{5}$$

where:

$\mathbf{\Phi}(f) = 2arcsin\sqrt{f_k}$ indicates the column vector (27×1 in the coming application in Section 3.1) of the transformed proportions of positive responses (successes);

$\tilde{\mathbf{Z}}$ is a matrix (27×7 in the application) in which the first column has been suppressed (*baseline*), for each of the three blocks of dummy variables, and it has been included in the column of the constant term;

$\tilde{\boldsymbol{\delta}}$ is a vector (27 × 1 in the application) of unknown coefficients;

$\boldsymbol{\varepsilon}$ is a vector (27× 1 in the application) of errors.

To estimate model (5) we used the ordinary least squares method (OLS).

The estimated probability values stay within the interval [0, 1].

3.1 APPLICATION

The overall evaluation, *Y*, toward the service provided by a university is expressed by 100 students on a dichotomous scale (1 = *satisfactory*, 0 = *unsatisfactory*), about 27 combinations of levels of judgment i = 1, 2, 3, (1 = *unsatisfactory*, 2 = *satisfactory*, 3 = *more than satisfactory*), with reference to attribute: *Teaching*, *Structures*, *Services* (see Section 2.1).

Considering the column vector 27×1 of the transformed proportions of positive response (success) and applying formula (5) we obtain the parameters estimate of the following model:

$$\hat{f} \cong \hat{p} = 0{,}326 + 1{,}255\ \tilde{z}_{12} + 1{,}587\ \tilde{z}_{13} + 0{,}436\ \tilde{z}_{22} + 0{,}611\ \tilde{z}_{23} + \ +0{,}438\ \tilde{z}_{32} + 0{,}612\ \tilde{z}_{33}.$$

In the application all coefficients are statistically significant at the α = 0.001 level.

We remark that the coefficients of model (5) are expressed in angular values: $\varphi = 2arcsin\sqrt{p_k}$. Consequently, to translate such coefficients in terms of experimental effects we need to transform the same in terms of proportions by the relation: $\hat{f}_k = sin^2(\frac{\varphi}{2})$.

The linear function probability values all fall in the interval [0, 1].

4. CONJOINT ANALYSIS WITH‘DICHOTOMOUS RESPONSE VARIABLE BY AN ADDITIVE BINARY CODING OF ORDINAL EXPERIMENTAL FACTORS: A PROPOSAL

Let Y denote the overall evaluation Y about a profile of the product expressed on a dichotomous scale (1 = satisfactory, 0 = unsatisfactory), on combinations of *ordered* levels (e.g.: 1 = *unsatisfactory*, 2 = *satisfactory*, 3 = *more than satisfactory*) (De Luca, 1999, p. 221) of the experimental factors.

A descriptive model of the result averages levels η_{hk}, which retains the ordered character of the experimental factors, in an experiment with, for instance, 2 factors with 3 levels (in absence of interactions) is:

$$\eta_{hk} = \mu + \alpha_h^{(1)} + \alpha_k^{(2)}, \qquad h, k = 1, 2, 3; \tag{6}$$

where:
μ is the parameter describing the average value;
the parameters $\alpha_h^{(1)}$, $\alpha_k^{(2)}$ denote the level factor effects computed as deviations from the “average value”.

With reference to model (6), the 9×7 matrix (of the full factorial design 3×3), that describes the experimental conditions, does not have complete characteristic, thus the parameters of (6) are not directly estimable (according to the principle of least squares).

A parameterization of the model with intercept descending from the following representation of the problem, is, in matrix symbolism:

$$\begin{bmatrix} \eta_{11} \\ \eta_{12} \\ \eta_{13} \\ \eta_{21} \\ \eta_{22} \\ \eta_{23} \\ \eta_{31} \\ \eta_{32} \\ \eta_{33} \end{bmatrix} = \begin{bmatrix} 10000 \\ 10010 \\ 10011 \\ 11000 \\ 11010 \\ 11011 \\ 11100 \\ 11110 \\ 11111 \end{bmatrix} \cdot \begin{bmatrix} \mu \\ \alpha_2^{(1)} \\ \alpha_3^{(1)} \\ \alpha_2^{(2)} \\ \alpha_3^{(2)} \end{bmatrix} \tag{7}$$

In (7) the reference level is η_{11}, the parameter $\alpha_2^{(1)}$ expresses the effect of the first factor, when this passes from the first to the second level, $\alpha_3^{(1)}$ indicates the *additional effect* that is added to the previous, when passing to the third level. A similar meaning, with reference to the second factor, have the parameters $\alpha_2^{(2)}$ and $\alpha_3^{(2)}$.

The choice probability vector of the overall judgment is estimated by a model of multiple linear regression of dummy variables.

Since the logistic cumulative distribution function $F(x)$ presents a linear behavior around the origin, i.e. for $x \in \{-1.39, 1.39\}$, interval corresponding approximately to values: $0.2 \leq F(x) \leq 0.8$, we propose a linear approximation of the probabilities p_j, $j = 1, 2, \ldots, J$ judges, with reference to the explanatory variables expressed on an ordinal scale.

Considering the heteroskedasticity of the disturbance term, we can avail of *Generalized Least Squares* (GLS) to estimate the parameters of the model.

Through empirical evidence it appears that by applying the procedure for estimating the parameters of a two-stage Goldberger method (1964, pp. 231-234), even if the qualitative experimental factors are represented with dummy variables (0,1): the *GLS estimators* coincide with the *Ordinary Least Squares* (OLS) estimators (De Luca, 2002).

Accordingly, the estimators of this linear model are correct and efficient.

But a distinct difficulty remains: the linear probability function formulation allows to fall outside the interval between 0 and 1.

If the [0,1] constraint is not spontaneously satisfied by sample estimates, we may truncate the p distribution in the following way: with p_{if} indicating the lowest value that p can take and p_{sup} its highest value, we assume $p_{inf} = 0$ e $p_{sup} = 1$.

4.1 APPLICATION

In the following application, concerning the degree of interest in different profiles of mobile phone, the experimental factors are observed on an ordinal scale.

In this study we are interested in how subjects evaluate various kinds of smartphones; $n = 79$ people took part in the COA study. The full factorial design of product profile is composed of 18 stimuli (i.e., 3×2×3 = 18).

They were asked to rate their preferences on a 1 to 10 equal interval scale for each (1= total disinterest in the profile; 10 = maximum interest) of the 18 hypothetical smartphone profiles.

The experimental factors, with the respective levels, are the following:

1) Weight: ≤ 94 grams, 95-105 grams, > 105 grams;
2) Battery run-time: ≤ 200 hours, > 200 hours;
3) price: ≤ 200€, 201-300€, > 300€.

The parameter estimates of the partial utility functions are obtained by multiple linear regression with ordinal predictors, expressed using an additive binary coding, unlike classical Coa (Green *et al.*, 1971), which is based on disjunctive binary coding of the experimental factors at issue.

The model:

$$\hat{Y}_j = 8{,}389 - 2{,}667\tilde{\tilde{z}}_{12j} - 1{,}000\tilde{\tilde{z}}_{13j} + 1{,}111\tilde{\tilde{z}}_{22j} - 1{,}833\tilde{\tilde{z}}_{32j} - 2{,}167\tilde{\tilde{z}}_{33j}$$

$j = 1, 2, \dots, n$.

5. A MULTICATEGORY RESPONS APPROACH TO CONJOINT ANALYSIS TO ESTIMATE MORE THAN ONE RESPONSE FUNCTION

The model provides as many overall desirability functions (aggregated part-worths sets), as the overall ordered categories, unlike the traditional *metric* and *nonmetric* Coa, which gives only one response function.

In the proposed model the multicategorical response variable (ordinal-polytomous), expressing overall desirability, is considered according to k ($k = 1, 2, \dots, K$) binary categories of response (i.e. for $K = 3$; 1 = *not desirable*, 2 = *desirable*, 3 = *most desirable*).

In this situation we have the levels of judgment on a three dimensional categorical scale, with dichotomous components $Y_{kj} \in \{0, 1\}$, $k = 1, 2, 3$, and to levels i of judgment $Z_{ij} \in \{0, 1\}$, which describe the choice of levels of explanatory factors.

To assess the value of effects on a response variable (overall) of the experimental factors, the dependent categorical variable is described as a function of dummy variables.

A dummy variable generalized multivariate linear regression model with parameter constraints can be used.

If we denote with p_{ksj} the probability that the respondent j-th ($j = 1, 2, .., J$) chooses the y_k category of attribute Y, associated with stimulus s ($s = 1, 2, \dots, S$), being the three response categories mutually exclusive, the model parameters are subject to the following constraints: $\sum_{k=1}^{3} p_{ks} = 1$; $0 \le p_{ks} \le 1$.

1) *First stage of calculation procedure: parameter estimation of univariate linear regressions*

Being K answers mutually exclusive and exhaustive thus much, the K-th equation is deduced by the remaining $q = K$-1 equations.

For the k-th category of the overall variable Y the univariate regression equation without intercept between variable Y_k and M factors, in matrix notation and in compact form is thus indicated:

$$\mathbf{Y}_k = \mathbf{Z}\,\boldsymbol{\delta}_k + \mathbf{E}_k, \quad k = 1, 2, \dots, K \qquad (8)$$

where:

$\mathbf{Y}_k$ = is the $SJ\times 1$ column vector, with generic observation Y_{ksj}, $k = 1, 2, \ldots, K$ (with $K = 2$ in the considered case), $s = 1, 2, \ldots, S$; $j = 1, 2, .., J$, associated with stimulus s, respondent j and category k of Y;

$\mathbf{Z}$ = fixed matrix of experimental design of dimensions $SJ\times\sum_{m=1}^{M} l_m$ (composed of J sub-matrices $\mathbf{Z}_j$ of indicator variables associated to S combinations of the plan, with elements $z_{ls}^{(m)}$, each one being a dummy variable representing to "l" level of "m" experimental factor in the "s" stimulus);

$\boldsymbol{\delta}_k$= column vector of order $\sum_{m=1}^{M} l_m \times 1$, with l_m indicating the number of levels of m-th factor (9×1 dimension in the considered case) containing the unknown parameters, $k = 1, 2, \ldots, q$; $m = 1, 2, \ldots, M$; $l = 1, 2, \ldots, l_m$, associated with the indicator variables of l levels of m attribute;

$\mathbf{E}_k = [E_{k11}, E_{k21}, \ldots, E_{kS1}, E_{k22}, \ldots, E_{kS2,\ldots,}E_{k1J}, \ldots, E_{kSJ}]'$; $SJ\times 1$ column vector (E_{ksj} indicates the observation error relevant to the s stimulus and the j-th statistical unit; E_{ksj} are *heteroskedastic* and *independent*) (Johnston, 2000; pp. 244 - 248).

The algebraic form of intercept model, in which the first column has been suppressed (*baseline*) for each of the blocks of dummy variables of the factors, and included in the column of the constant term, is:

$$y_{ks} = \tilde{c}_k + \sum_{m=1}^{M} \sum_{l=2}^{l_m} \tilde{\delta}_{kl}^{(m)} \tilde{z}_{ls}^{(m)} + E_{ks}, \quad k = 1, \ldots, q; s = 1, 2, \ldots, S \qquad (9)$$

were:

$\tilde{c}_k$ = proportion of cases with value 1 for Y_k variable pertaining to the reference category (*baseline*).

2) *Second stage of the calculation procedure: parameter estimation of univariate linear regressions in compact form*

The $\tilde{\mathbf{Z}}$ matrix of the experimental design is of order $S\times(1 + \sum_{m=1}^{M} l_m - M)$.

The q equations Y_k equation (9) in compact form are expressed as follows:

$$\mathbf{Y}^* = \tilde{\mathbf{Z}}^* \tilde{\boldsymbol{\delta}}^* + \mathbf{E}^* \qquad (10)$$

were:

$\mathbf{Y}^*$ = compound vector $vec(\mathbf{Y}_1, \mathbf{Y}_2)$ is the vector obtained by arranging in columns the elements y_1 and y_2 of the vectors $\mathbf{Y}_1$ and $\mathbf{Y}_2$, respectively, for each of $J = 100$ respondents on S stimuli;

$\tilde{\mathbf{Z}}^*$ is a square compound diagonal matrix, containing $q\times q$ submatrices with $JS\times(1 + \sum_{m=1}^{M} l_m - M)$ dimensions, of which the q submatrices $\tilde{\mathbf{Z}}$ positioned on the main diagonal (equal among themselves) present in column the independent indicator variables corresponding to the different equations, while the remaining submatrices are composed of null elements;

$\widetilde{\boldsymbol{\delta}}^*$ is a compound vector of the q column vectors of the regression coefficients each of order $(1+\sum_{m=1}^{M} l_m - M)\times 1$;

$\mathbf{E}^*$ is compound vector (Vec) of q column vectors of $\mathbf{E}_k$ errors.

With reference to (10), interpreted in probabilistic terms ("average proportion of *l*'s"), the condition of inequality which is to be subject to a probability value is: $0 \leq p_{ks} \leq 1$ such condition is translated in the constraint:

$$0 \leq \boldsymbol{z}_j' \boldsymbol{\delta}_k \leq 1 \qquad k = 1, 2, \ldots, q; \quad s = 1, 2, \ldots, S. \tag{11}$$

Constraint (11) imposes, for the parameter estimation of model (9), the use of constrained least squares method and quadratic programming on *each* univariate equation.

In compact form the multiple regression multivariate model, considering only the first two categories of *Y* is represented in the following way:

$$\mathbf{Y}^* = \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{Z}} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{Z}} \end{bmatrix} \cdot \begin{bmatrix} \widetilde{\boldsymbol{\delta}}_1 \\ \widetilde{\boldsymbol{\delta}}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{E}_1 \\ \mathbf{E}_2 \end{bmatrix} = \mathbf{H}\widetilde{\boldsymbol{\delta}}^* + \mathbf{E}^*$$

where:

$\mathbf{Y}^* = \text{vec}(\mathbf{Y}_1, \mathbf{Y}_2)$

$\mathbf{H} = \mathbf{I} \otimes \widetilde{\boldsymbol{Z}}^*$, where $\mathbf{I}$ is a 2×2 identity matrix and $\otimes$ indicates a Kronecker product;

$\widetilde{\boldsymbol{\delta}}^* = \text{vec}(\widetilde{\boldsymbol{\delta}}_1, \widetilde{\boldsymbol{\delta}}_2)$

$\mathbf{E}^* = \text{vec}(\mathbf{E}_1, \mathbf{E}_2)$.

3) *Third stage of the calculation procedure: the constrained generalized multivariate regression model*

In order to estimate the parameters of the multivariate model (10) we consider the covariance between Y_k, $k = 1, 2, \ldots, q$.

Considering that for each respondent j ($j = 1, 2, \ldots, J$) the Y_{kj} valuation is described by a multinomial observation with q components and that there is a stochastic independence on varying of j, to estimate the multivariate model parameters we need to consider a $qn \times qn$ dimension variance-covariance matrix $\mathbf{\Phi}$ (the variance-covariance matrix of Y* vector among the Y_k), with elements (Kotz and Johnson, 1985): $\text{Var}(Y_{ksj}) = p_{ksj}(1-p_{ksj})$, were: p_{ksj} is the probability for a j respondent to choose the k category for the s combination; $\text{Cov}(Y_{ksi}, Y_{qsj}) = - p_{ksj}\ p_{qsj}$ (De Luca *et al.*, 2006).

The estimates of the elements of matrix $\mathbf{\Phi}$ are calculated on the basis of estimations $\hat{p}_{ksj}$ obtained by performing the *Generalized Least Squares*-GLS (Goldberger, 1964) method separately on each equation of the model as estimates in the second stage.

To estimate the multivariate linear regression model (10) performed by GLS method we minimize the following mathematical expression (where $\widehat{\mathbf{\Phi}}^{-1}$ is inverse matrix of the $\widehat{\mathbf{\Phi}}$) under the condition

$$F = (\mathbf{Y}^* - \tilde{\mathbf{Z}}^*\tilde{\boldsymbol{\delta}}^*)'\widehat{\mathbf{\Phi}}^{-1}\left(\mathbf{Y}^* - \tilde{\mathbf{Z}}^*\tilde{\boldsymbol{\delta}}^*\right) = \textit{minimum} \qquad (12)$$

Formula (12), joined to the constraint (11) and to the condition $\sum_{k=1}^{3} \tilde{\mathbf{z}}_j' \, \tilde{\boldsymbol{\delta}}_k = 1$, entails the use of quadratic programming.

5.1 APPLICATION

The model was applied to the overall desirability evaluations expressed on the $K = 3$ categories: "undesirable", "desirable", "most desirable", by a sample of $J = 100$ insurance officers (homogeneous respondents) on $S = 24$ profiles of an insurance policy. The $M = 4$ attributes were: X_1 = "policy duration" (with levels: 5, 8 years); X_2 = "minimum denomination" (2,500 €, 5,000 €); X_3 = "stock exchange index" (Ftse/Mib, Dow Jones, Nikkei); X_4 = "service at expiry" (paid-up capital, income for life).

To estimate the parameters (see Table 1) of response functions (formula 12) we used the *Constrained Non Linear Regression* program.

The full factorial design of product profile, with *casualization* of four factors and levels of the index-linked life policy, is composed of 24 stimuli (i.e., 2×2×3×2 = 24), with uniform replications design.

Tab. 1: Estimates of three sets of the aggregated part-worths utilities of the Coa model

Overall category		Estimated coefficient of the 1st equation		Overall category	Estimated coefficient of the 2nd equation		Overall category	Estimated coefficient of the 3rd equation	
"undesirable"	*Baseline*	$\tilde{\delta}_1$	0,430	"desirable"	$\tilde{\delta}_2$	0,366	"most desirable"	$\tilde{\delta}_3$	0,204
	8 years	$\tilde{\delta}_{12}^{(1)}$	0,024		$\tilde{\delta}_{22}^{(1)}$	-0,009		$\tilde{\delta}_{32}^{(1)}$	-0,015
	5,000 €	$\tilde{\delta}_{12}^{(2)}$	-0,006		$\tilde{\delta}_{22}^{(2)}$	0,004		$\tilde{\delta}_{32}^{(2)}$	0,003
	Dow J.	$\tilde{\delta}_{12}^{(3)}$	-0,332		$\tilde{\delta}_{22}^{(3)}$	0,279		$\tilde{\delta}_{32}^{(3)}$	0,053
	Nikkei	$\tilde{\delta}_{13}^{(3)}$	-0,340		$\tilde{\delta}_{23}^{(3)}$	0,256		$\tilde{\delta}_{33}^{(3)}$	0,083
	Income	$\tilde{\delta}_{12}^{(4)}$	0,196		$\tilde{\delta}_{22}^{(4)}$	-0,096		$\tilde{\delta}_{32}^{(4)}$	-0,010

Tab. 2: Comparison between observed frequencies (f_{ks}) and probability values ($\hat{P}_{ks}$) estimated through the multivariate linear model, for the overall categories

Frequency and probability / Stimulus	Overall category: "undesirable" (Y_1)		Overall category: "desirable" (Y_2)		Overall category:" "most desirable" (Y_3)	
s	f_{1s}	$\hat{P}ks$	f_{2s}	$\hat{P}ks$	f_{3s}	$\hat{P}ks$
1	0,45	0,43	0,45	0,37	0,10	0,20
16	0,10	0,09	0,61	0,65	0,29	0,26
9	0,08	0,11	0,60	0,62	0,32	0,28
18	0,10	0,12	0,60	0,64	0,30	0,24
12	0,05	0,08	0,67	0,63	0,28	0,29
19	0,52	0,45	0,34	0,36	0,14	0,19
10	0,08	0,12	0,54	0,64	0,38	0,24
24	0,12	0,11	0,48	0,61	0,40	0,27
2	0,10	0,10	0,50	0,65	0,40	0,26
13	0,50	0,42	0,43	0,37	0,07	0,21
5	0,51	0,45	0,41	0,36	0,08	0,19
17	0,10	0,09	0,55	0,62	0,35	0,29
6	0,35	0,31	0,54	0,54	0,11	0,15
21	0,31	0,31	0,54	0,52	0,15	0,17
15	0,31	0,29	0,60	0,55	0,09	0,16
7	0,59	0,63	0,26	0,27	0,15	0,10
23	0,32	0,28	0,57	0,53	0,11	0,19
8	0,27	0,29	0,62	0,55	0,11	0,16
14	0,29	0,29	0,53	0,53	0,18	0,19
11	0,33	0,32	0,59	0,54	0,08	0,14
20	0,63	0,65	0,33	0,26	0,04	0,09
4	0,34	0,30	0,52	0,52	0,14	0,18
3	0,65	0,64	0,22	0,26	0,13	0,09
22	0,63	0,62	0,24	0,27	0,13	0,11

In order to empirically asses the predictive capacity of the estimated model, Table 2 shows the probabilities estimated for all the modality combination (experimental condition "s") of the factors and the corresponding values of the observed proportions.

We observe a satisfactory model fitting, as the predicted probabilities turn out to be very near the corresponding proportions for all the modality combinations of the experimental design.

The analysis model here proposed provides two main advantages:

1) the use of probability as an average response, which does not require scale adjustments to render the preference scale as "metric";
2) a cross-check of the level effects on the *k* categories.

6. SUMMARY TABLE OF THE APPROACHES TO FULL-PROFILE CONJOINT ANALYSIS BY MULTIPLE *LINEAR* REGRESSION ANALYSIS PRESENTED HERE

Table 3 shows the summary of all models characteristics of conjoint modeling by multiple *linear* regression analysis so far submitted.

Tab. 3: Characteristics of conjoint modeling by multiple linear regression analysis

Alternative approaches: / *Characteristic*	*1. COA with dichotomous dependent variable:*	*2. COA with dichotomous dependent variable*	*3. COA with dichotomous and limited dependent variables*	*4. COA to estimate more than on response function*
1.Scaling of the response data	Category assignment	Category assignment	Category assignment	Multicategory assignment
2. Level of response aggregation	Aggregated level	Aggregated level	Individual level	Individual level
3.Estimation method	Weighted least squares	Arcsine transfor-mation	Constrained generalized least squares	Constrained generalized multivariate linear multiple regression

B) Some approaches to full-profile conjoint analysis by multiple logistic regression analysis

7. ORDINAL LOGISTIC REGRESSION FOR THE ESTIMATION OF THE RESPONSE FUNCTIONS THROUGH CONJOINT ANALYSIS

In this Coa model we assume that the respondent evalua-tive judgement Y_k on the overall desirability consists in a choice (rather than *rating* or to *ranking* product profiles) of one of the ordered k (k = 1, 2,…, K) desirability categories on ordinal scale 1 - 5 (1 = “least desirable”, 5 = “most desirable”) for each S hypothetical product profiles.

The ordinal response variable is estimated by an *ordered logit model*, that directly incorporates the order of Y_k categories.

To link the categories of overall evaluation Y_k to the factor levels, we adopt a cumulative logit model at the aggregated level (*pooled model*).

The novelty value in this approach is that one set of aggregated part-worths (response function) is estimated in connection with *each* category Y_k, as many k as the K overall ordered categories are, unlike the traditional metric and nonmetric COA and Choice Based Conjoint (CBC) analysis, which give only one response function (only *one set* of aggregated part-worths).

To obtain *univocal* estimates of the parameters the first variable of each set of the dummy exploratory variables is dropped ($\tilde{\mathbf{z}}_S$).

The effects of the factors express the variations of the probabilities P_{ks} associated with the vector $\tilde{\mathbf{z}}_S$ (the vector of the dummy explanatory variables relative to the profile s, s = 1, 2, …, S):

$$P(Y_k = 1|\tilde{\mathbf{z}}_s) = \frac{\exp(\delta_k + \boldsymbol{\delta}'\tilde{\mathbf{z}}_s)}{1+\exp(\delta_k + \boldsymbol{\delta}'\tilde{\mathbf{z}}_s)} = F_k(\tilde{\mathbf{z}}_s) \quad (k = 1, \ldots, K) \tag{13}$$

where:

$\tilde{\mathbf{z}}_s$ is the vector of the reduced matrix $\tilde{\mathbf{Z}}$ (see Section 5).

δ_k is the constant term associated to the reference category. The *cutpoint parameters* are not decreasing in k, since the cumulative logit is an increasing function of $F_k(\tilde{\mathbf{z}}_s)$, which is itself increasing in k for fixed $\tilde{\mathbf{z}}_s$;

$\tilde{\boldsymbol{\delta}}'$ is the vector of the unknown coefficients and does not have a k subscript; it is known also as *parallel regression,* because there is an identical effect of each indicator variables for all K-1 dichotomous responses (*Proportional Odd Assumption*); so, the model assumes the same effects as $\tilde{\mathbf{Z}}$ for all K-1 on all *cumulative logit* results, in a *parsimonious* model for ordinal data (when this model fits well it requires a single parameter, rather than K-1 parameters, to describe the effect of $\tilde{\mathbf{z}}$).

To estimate such probabilities we use an *aggregate* level model across the J homogeneous respondents, whose evaluations, on each product profile, are considered J repeated observations.

Therefore the k-th cumulative response probability is:

$$P_{ks}(Y \leq k|\tilde{\mathbf{z}}_s) = F_k(\tilde{\mathbf{z}}_s) = \pi_1(\tilde{\mathbf{z}}_s) + \pi_2(\tilde{\mathbf{z}}_s) + \cdots + \pi_k(\tilde{\mathbf{z}}_s);\ k = 1, 2, \ldots, K \tag{14}$$

where $\pi_k(\tilde{\mathbf{z}}_s)$ is the probability of response k associated with vector $\tilde{\mathbf{z}}_s = [1, z_{12}, z_{13}, \ldots, z_{M_{l_M}}]$.

The cumulative probabilities reflect the ordering, with:

$$P_{ks}(Y \leq 1|\tilde{\mathbf{z}}_s) \leq P_{ks}(Y \leq 2|\tilde{\mathbf{z}}_s) \cdots P_{ks}(Y \leq K|\tilde{\mathbf{z}}_s), \text{ and: } P_{ks}(Y \leq K|\tilde{\mathbf{z}}_s) = 1.$$

In the model the Kth equation can be obtained from the remaining $q = K$-1.

The cumulative logits of the (K-1) cumulative probabilities are:

$$L_k(\tilde{\mathbf{z}}_s) = \text{logit}[F_k(\tilde{\mathbf{z}}_s)] = \ln\left[\frac{F_k(\tilde{\mathbf{z}}_s)}{1-F_k(\tilde{\mathbf{z}}_s)}\right] = l_n\left[\frac{\pi_1(\tilde{\mathbf{z}}_s)+\pi_2(\tilde{\mathbf{z}}_s)+\cdots\pi_k(\tilde{\mathbf{z}}_s)}{\pi_{k+1}(\tilde{\mathbf{z}}_s)+\pi_{k+2}(\tilde{\mathbf{z}}_s)+\cdots\pi_K(\tilde{\mathbf{z}}_s)}\right] = \delta_k + \tilde{\boldsymbol{\delta}}'\tilde{\mathbf{z}} \tag{15}$$

with $k = 1, 2, \ldots, K$-1.

7.1 THE APPLICATION: DESIRABILITY OF MOBILE PHONES

The model was applied to the overall desirability evaluations expressed on the $K = 5$ categories (expressed by disjunctive binary coding) by a sample of $J = 79$ users on $S = 18$ new profiles of *mobile phones*, related to a full-factorial experimental design.

The $M = 3$ experimental factors (attributes) and levels were:

X_1 = "weight"; levels: ≤ 94grams, 95-105 grams, > 105 grams;

X_2 = "autonomy"; levels: ≤ 200 h, > 200 h;

X_3 = "price"; levels: 200 €, 200-300 €, > 300 €.

The model is estimated by *PLUM-Ordinal regression procedure*, available in *SPSS*. The parameters (Table 4) were estimated using the maximum likelihood method linked to the *Proportional Odd Assumption* hypothesis (the Fisher's Scoring optimisation algorithm) (Table 4). The judgement evaluations are pooled across respondents (*pooled model*) (De Luca, 2011).

Intercepts values result increasing since the model is cumulative. In the application all coefficients are statistically significant at the $\alpha = 0.001$ level.

In order to empirically asses the predictive capacity of the estimated model, Table 5 shows the probabilities estimated for all the level combinations (experimental conditions "s") of the factors and the corresponding values of the observed proportions.

We observe a satisfactory model fitting, as the predicted probabilities turn out to be very near the corresponding proportions for all the modality combinations of the experimental design.

Tab. 4: Estimates of four set of the aggregated part-worths utilities of the Coa model ordinal logistic regression

	Equations	*Estimated coefficient*	*Standard error*	*df*	*Wald c2*	*p-value*
Intercept	$Y = 1$	–4.409	0.171	1	662.647	0,000
	Y = 2	–2.600	0.144	1	327.567	0,000
	Y = 3	–0.610	0.126	1	23.470	0,000
	Y = 4	1.512	0.140	1	116.584	0,000
Factor	*Levels*					
Weight	z_{12}	–0.944	0,122	1	59.439	0,000
	z_{13}	–1.930	0,129	1	223,215	0,000
Autonony	z_{22}	1.041	0,101	1	105,829	0.000
Price	z_{32}	–1.197	0,124	1	93,015	0,000
	z_{32}	–2.355	0,134	1	310,179	0,000

The here proposed model provides also the following remarkable advantages:

1) the use of probability $\hat{p}_{ks}$ as an *average response*, which does not require scale adjustments to render the preference scale as “metric”;
2) the estimate of one set of aggregated part-worths in connection with *each category k*;
3) a cross-check of the attribute level effects on the different k categories of Y_k.

Tab. 5: Comparison of the probabilities estimated through Conjoint analysis model, and the corresponding proportions for all the modality combinations of the experimental design

s	Y_1		Y_2		Y_3		Y_4		Y_5	
	Proba-bility	*Propor-tion*	*Proba-bility*	*Propor-tion*	*Proba-bility*	*Propor-tion*	*Proba-bility*	*Propor-tion*	*Proba-bility*	*Propor-tion*
	$\tilde{\pi}_1(\mathbf{z}_s)$	p_1	$\tilde{\pi}_2(\mathbf{z}_s)$	p_2	$\tilde{\pi}_3(\mathbf{z}_s)$	p_3	$\tilde{\pi}_4(\mathbf{z}_s)$	p_4	$\tilde{\pi}_5(\mathbf{z}_s)$	p_5
1	0,01	0,01	0,06	0,04	0,28	0,30	0,47	0,46	0,18	0,19
2	0,04	0,04	0,16	0,20	0,45	0,41	0,29	0,30	0,06	0,05
3	0,11	0,13	0,33	0,29	0,41	0,46	0,13	0,11	0,02	0,01
4	0,00	0,00	0,02	0,01	0,14	0,14	0,45	0,44	0,38	0,41
5	0,01	0,03	0,07	0,06	0,31	0,25	0,45	0,49	0,16	0,16
6	0,04	0,06	0,17	0,15	0,45	0,49	0,28	0,24	0,06	0,05
7	0,03	0,01	0,13	0,16	0,42	0,39	0,34	0,35	0,08	0,08
8	0,09	0,08	0,29	0,28	0,43	0,49	0,15	0,14	0,03	0,01
9	0,25	0,24	0,42	0,39	0,27	0,30	0,06	0,06	0,01	0,00
10	0,01	0,00	0,05	0,06	0,27	0,24	0,47	0,51	0,20	0,19
11	0,04	0,04	0,15	0,14	0,44	0,47	0,31	0,33	0,07	0,03
12	0,10	0,13	0,31	0,33	0,42	0,35	0,14	0,19	0,02	0,00
13	0,03	0,03	0,12	0,14	0,42	0,44	0,35	0,29	0,08	0,10
14	0,09	0,04	0,29	0,33	0,44	0,46	0,16	0,14	0,03	0,04
15	0,24	0,22	0,42	0,44	0,28	0,24	0,06	0,09	0,01	0,01
16	0,08	0,08	0,26	0,29	0,45	0,44	0,18	0,13	0,03	0,06
17	0,22	0,23	0,41	0,41	0,30	0,28	0,07	0,06	0,01	0,03
18	0,47	0,49	0,37	0,29	0,13	0,18	0,02	0,03	0,00	0,01

8. MULTIVARIATE LOGISTIC REGRESSION FOR THE ESTIMATION OF RESPON-SE FUNCTIONS IN CONJOINT ANALYSIS

In this Coa model the polytomous response variable is described by a sequence of binary variables. The model provides as many overall desirability functions (aggregated part-worths sets), as the overall ordered categories are.

We assume that the respondent’s evaluative judgement on the *overall* desirability is expressed on each profile of the new product, consisting in a choice of one of the K desirability categories.

To link the *overall* desirability (ordinal dependent variable Y, with levels: Y_k, $k = 1, 2, \dots, K$) with the levels of the M experimental factors, the summarizing vector of the choice probability of one of the mentioned K ordered categories is interpreted via a *multivariate multiple logistic regression model.*

We denote by y_{ksj} the desirability category k of the concept s for the respondent j.

The factor effects express the variations of the probabilities p_{ks} associated with vector $\mathbf{z}_s$ corresponding to the combination s ($s = 1, 2, \dots, S$) of M factor levels, as follows:

$$p(Y_k = 1 \mid \mathbf{z}_s) = \pi_k(\mathbf{z}_s) = \exp(\delta_{k0} + \delta'_{k1}\,\mathbf{z}_s)\,/[1 + \exp(\delta_{k0} + \delta'_{k1}\,\mathbf{z}_s)] \quad (16)$$

where:

$\boldsymbol{\delta}' = (\delta_0, \boldsymbol{\delta}'_1)$ is the unknown vector of regression coefficients of the predictor variables;

$\mathbf{z}_s$ is the vector of the dummy factors relevant to the combination, or concept s.

To estimate such probabilities $\pi_k(\mathbf{z}_s)$, we use an *aggregated* level model across the J homogeneous research respondents (we consider the J respondents as if they were repeated observations).

To estimate the relation between the Y_k ($k = 1, 2, \dots, K$) dependent variable and $m = 1, 2, \dots, M$, predictive factors, with levels $l = 1, 2, \dots, l_m$, the K *overall* categories (Y_k,) are codified as K dummy variables; also the independent variables are codified as dummy variables (Z).

In this multivariate model the Kth equation can be deduced from the remaining $q = K$-1 equations.

To solve the linear dependency among all the independent variables the model is reparametrized using $z_1^{(m)}$ as a reference category, and the model with intercept is:

$$g_k(\tilde{\mathbf{z}}_s) = \tilde{\delta}_{k0} + \sum_{m=1}^{M} \sum_{l=2}^{l_m} \tilde{\delta}_{kl}^{(m)}\, \tilde{z}_{lsj}^{(m)} + e_{ksj}, \quad (17)$$

where:

$g_k(\tilde{\mathbf{z}}_s)$ is the logit of the sth profile with regard to the kth dependent variable;

$\tilde{\delta}_{k0}$ is a constant term;

$\tilde{\delta}_{kl}^{(m)}$ is the unknown regression coefficient for the lth level of the m factor;

$\tilde{z}_{lsj}^{(m)}$ is the dummy variable for the lth level of the m factor in the combination s;

e_{ksj} is the error term pertinent to the stimulus s and subject j ($j = 1, 2, \dots, J$).

Indicating with $\tilde{\mathbf{Z}}$ the design matrix in equation (17), the q equations $g_k(\tilde{\mathbf{z}}_s)$ can be compactly expressed as follows:

$$\mathbf{g}^* = \tilde{\mathbf{Z}}^* \tilde{\boldsymbol{\delta}}^*, \quad (18)$$

where:
$\mathbf{g}^*$ is a compound vector (*vec*) of q column vectors $g_k(\tilde{\mathbf{z}})$;
$\tilde{\mathbf{Z}}^*$ is a square compound diagonal matrix, containing $q \times q$ submatrices $\tilde{\mathbf{Z}}$;
$\tilde{\boldsymbol{\delta}}^*$ is a compound vector of the q column vectors $\tilde{\boldsymbol{\delta}}_k$.

To estimate the multivariate model parameters of equation (18) we need to consider the variance-covariance matrix $\boldsymbol{\Phi}$, between the Y_k, with elements $\mathrm{Var}(Y_{ksj}) = p_{ksj}\,(1 - p_{ksj})$, where: p_{ksj} is the probability for a jth respondent to choose category k for the combination s; $\mathrm{Cov}(Y_{ksi}, Y_{ksj}) = - p_{ksj}\, p_{qsj}$.

The estimates of the $\boldsymbol{\Phi}$ matrix elements are calculated on the basis of estimations $\hat{p}_{ksj}$, obtained by performing a logistic regression analysis separately on each dependent variable, using the maximum likelihood method to each equation (17).

To estimate the multivariate logistic regression model (18) we minimize the following mathematical expression (where $\hat{\boldsymbol{\Phi}}^{-1}$ is inverse matrix of the $\hat{\boldsymbol{\Phi}}$):

$$F = (\mathbf{g}^* - \tilde{\mathbf{Z}}^*\tilde{\boldsymbol{\delta}}^*)'\, \hat{\boldsymbol{\Phi}}^{-1}\, (\mathbf{g}^* - \tilde{\mathbf{Z}}^*\tilde{\boldsymbol{\delta}}^*)' \tag{19}$$

8.1 THE APPLICATION: DESIRABILTY OF INSURANCE POLICIES

The model was applied to the overall desirability evaluations expressed in the K = 3 categories: "undesirable", "desirable", "most desirable", by a sample of J = 100 insurance officers (homogeneous respondents) on S = 24 profiles of the insurance policy.

The M = 4 attributes were: X_1 = "policy duration" (with levels: 5, 8 years); X_2 = "minimum denomination" (2,500 €, 5,000 €); X_3 = "stock exchange index" (*Ftse/Mib*, *Dow Jones*, *Nikkei*); X_4 = "service to expiry" (paid-up capital, income for life). To estimate the parameters of the response functions of function (19) we used the *Constrained Non Linear Regression* program of the SPSS software (see Table 6).

Tab. 6: Estimates of three sets of the aggregated part-worths utilities of the COA logistic regression model

Levels	Overall category	Estimated coefficient of the 1st equation		Overall category	Estimated coefficient of the 2nd equation		Overall category	Estimated coefficient of the 3rd equation	
Baseline		$\tilde{\delta}_1$	-0,21		$\tilde{\delta}_2$	-0,71		$\tilde{\delta}_3$	-1,25
8 years		$\tilde{\delta}_{12}^{(1)}$	0,05		$\tilde{\delta}_{22}^{(1)}$	-0,01		$\tilde{\delta}_{32}^{(1)}$	-0,06
5,000 €	"unde-sirable	$\tilde{\delta}_{12}^{(2)}$	-0,36	"desira-ble"	$\tilde{\delta}_{22}^{(2)}$	0,14	"most desir-able"	$\tilde{\delta}_{32}^{(2)}$	0,30
Dow J.		$\tilde{\delta}_{12}^{(3)}$	-1,34		$\tilde{\delta}_{22}^{(3)}$	1,12		$\tilde{\delta}_{32}^{(3)}$	0,01
Nikkei		$\tilde{\delta}_{13}^{(3)}$	-1,41		$\tilde{\delta}_{23}^{(3)}$	1,07		$\tilde{\delta}_{33}^{(3)}$	0,13
Income"		$\tilde{\delta}_{12}^{(4)}$	0,65		$\tilde{\delta}_{22}^{(4)}$	-0,20		$\tilde{\delta}_{32}^{(4)}$	-0,90

Tab. 7: Comparison between observed frequencies (f_{ks}) and probability values ($\hat{p}_{ks}$) estimated through the multivariate logistic model, for the overall categories

Frequency and probability / Stimulus	Overall category: "undesirable" (Y_1)		Overall category: "desirable" (Y_2)		Overall category:"most desirable" (Y_3)	
s	f_{1s}	$\hat{p}_{ks}$	f_{2s}	$\hat{p}_{ks}$	f_{3s}	$\hat{p}_{ks}$
1	0,45	0,44	0,45	0,35	0,10	0,22
16	0,10	0,13	0,61	0,64	0,29	0,26
9	0,08	0,13	0,60	0,59	0,32	0,30
18	0,10	0,14	0,60	0,62	0,30	0,27
12	0,05	0,12	0,67	0,62	0,28	0,29
19	0,52	0,46	0,34	0,32	0,14	0,22
10	0,08	0,18	0,54	0,60	0,38	0,21
24	0,12	0,17	0,48	0,58	0,40	0,24
2	0,10	0,17	0,50	0,63	0,40	0,20
13	0,50	0,36	0,43	0,36	0,07	0,28
5	0,51	0,38	0,41	0,33	0,08	0,28
17	0,10	0,16	0,55	0,61	0,35	0,23
6	0,35	0,24	0,54	0,56	0,11	0,13
21	0,31	0,29	0,54	0,52	0,15	0,11
15	0,31	0,22	0,60	0,59	0,09	0,13
7	0,59	0,60	0,26	0,30	0,15	0,10
23	0,32	0,22	0,57	0,56	0,11	0,15
8	0,27	0,28	0,62	0,57	0,11	0,09
14	0,29	0,27	0,53	0,55	0,18	0,11
11	0,58	0,55	0,22	0,28	0,20	0,14
20	0,33	0,30	0,59	0,54	0,08	0,10
4	0,34	0,23	0,52	0,53	0,14	0,15
3	0,63	0,62	0,33	0,27	0,04	0,10
22	0,59	0,53	0,24	0,31	0,17	0,14

In order to empirically asses the predictive capacity of the estimated model, Table 7 shows the probabilities estimated for all the modality combinations of the factors and the corresponding values of the observed proportions.

We observe a satisfactory model fitting, as the predicted probabilities turn out to be near the corresponding proportions for all the level combinations of the experimental design.

The *most desirable* profile is the stimulus s =12 (Policy duration: *5 years*; Minimum denomination: *5,000€*; Stock exchange index: *Down Jones*; Service to expiry: *paid-up capital*); the *least desirable profile* is the stimulus s = 3 (Policy duration : *8 years*; Minimum denomination: *2,500€*; Stock exchange index: *Ftse/Mib*; Service to expiry: *income for life*).

9. LOGISTIC REGRESSION RESPONSE WITH MAIN AND INTERACTION EFFECTS

In this model the polytomous response variable (i.e. evaluation of the *overall desirability* of alternative product profiles) is described by a sequence of binary variables.

To link the categories of overall evaluation to the factor levels, we adopt – at the aggregated level – a multivariate logistic regression model, based on a main and two-factor interaction effects experimental design.

The model provides as many overall desirability functions (aggregated part-worths sets), as the overall ordered categories are.

The model is reparametrized using as reference category (see Section 8). The algebraic form of the response functions with *main and first-order interaction* effects is:

$$g_k(\tilde{\mathbf{z}}_s) = \tilde{\delta}_{k0} + \sum_{m=1}^{M} \sum_{l=2}^{l_m} \tilde{\delta}_{kl}^{(m)} \tilde{z}_{ls}^{(m)} + \sum_{m=1}^{M} \sum_{l=2}^{l_m} \sum_{h=2}^{h_p} \tilde{\delta}_{klh}^{(m,p)} \tilde{z}_{lhs}^{(m,p)} + e_{ks} \qquad (20)$$

$k = 1, \ldots, q$ (see Section 8); $s = 1, 2, \ldots, S$; $h = 1, 2, \ldots,$; $p = m + 1, m + 2, \ldots, M$;

where:

$\tilde{g}_k(\tilde{\mathbf{z}}_s)$ is the logit of the sth profile with regard to the kth dependent variable;

$\tilde{\delta}_{k0}$ is the constant term;

$\tilde{\delta}_{kl}^{(m)}$ is the unknown regression coefficient for the lth level of factor m;

$\tilde{z}_{ls}^{(m)}$ is the dummy variable for the lth level of factor m in the combination s.

$\tilde{z}_{lhs}^{(m,p)}$ is the dummy variable correspondent to l level of factor m and to the h level of factor p in the stimulus s;

e_{ks} is the error term pertinent to the stimulus s.

The q equations $g_k(\tilde{\mathbf{z}}_s)$ can be expressed compactly as follows:

$$\mathbf{g}^* = \tilde{\mathbf{Z}}^* \tilde{\boldsymbol{\delta}}^* \qquad (21)$$

where:

$\mathbf{g}^*$ is a compound vector of q column vectors $g_k(\tilde{\mathbf{z}})$;

$\tilde{\mathbf{Z}}^*$ is a square compound diagonal matrix, containing $q \times q$ sub-matrices $\tilde{\mathbf{Z}}$ (the $\tilde{\mathbf{Z}}$ denotes the design matrix of equation (20) ;

$\tilde{\boldsymbol{\delta}}^*$ is a compound vector of the q column vectors $\tilde{\boldsymbol{\delta}}_k$.

To estimate the multivariate model parameters we need to consider the variance-covariance matrix $\mathbf{\Phi}$. The estimates of the $\mathbf{\Phi}$ matrix elements are calculated on the basis of estimations $\hat{p}_{ksj}$ obtained by performing logistic regression analysis separately on each dependent variable, using the maximum likelihood method to each equation (21).

To estimate the multivariate logistic regression model we minimize the following mathematical expression:

$$F = (\mathbf{g}^* - \tilde{\mathbf{Z}}^* \tilde{\boldsymbol{\delta}}^*)' \hat{\mathbf{\Phi}}^{-1} (\mathbf{g}^* - \tilde{\mathbf{Z}}^* \tilde{\boldsymbol{\delta}}^*). \quad (22)$$

where $\hat{\mathbf{\Phi}}^{-1}$ is the inverse matrix of the $\hat{\mathbf{\Phi}}$.

To estimate the parameters (part-worths) of the response functions of equation (22) was used the Constrained Non Linear Regression program of the SPSS software.

9.1 THE APPLICATION: DESIRABILITY OF INSURANCE POLICIES

The model was applied to the overall desirability evaluations expressed on K = 3 ordinal categories: "undesirable", "desirable", "most desirable", by a sample of J = 100 insurance officers (homogeneous respondents) on S = 24 profiles of the insurance policy (see Section 8.1).

The M = 4 attributes were: X_1 = "policy duration" (with levels: 5, 8 years); X_2 = "minimum denomination" (2,500 €, 5,000 €); X_3 = "stock exchange index" (*Ftse/Mib*, *Dow Jones*, *Nikkei*); X_4 = "service to expiry" (*paid-up capital*, *income for life*).

To estimate the parameters of the response functions of equation (22) was used the *Constrained Non Linear Regression* program of the SPSS software.

Tab. 8: Full factorial design with restricted casualisation of four factors and factor levels of the index-linked life policy

Stimulus (s)	Policy duration (years) (X_1)	Minimum denomination (euro) (X_2)	Stock exchange index (X_3)	Service to expiry (X_4)
1	5	2500	Nikkei	paid-up capital
16	5	5000	Comit	paid-up capital
9	8	5000	Dow Jones	paid-up capital
18	8	5000	Comit	paid-up capital
12	5	5000	Dow Jones	paid-up capital
19	8	2500	Nikkei	paid-up capital
10	8	2500	Comit	paid-up capital
24	8	2500	Dow Jones	paid-up capital
2	5	2500	Comit	paid-up capital
15	5	5000	Nikkei	paid-up capital
5	8	5000	Nikkei	paid-up capital
17	5	2500	Dow Jones	paid-up capital
6	8	5000	Comit	income for life
21	8	2500	Dow Jones	income for life
7	5	5000	Comit	income for life
23	5	2500	Nikkei	income for life
8	5	5000	Dow Jones	income for life
14	5	2500	Comit	income for life
11	5	2500	Dow Jones	income for life
20	8	5000	Nikkei	income for life
4	8	2500	Nikkei	income for life
13	8	5000	Dow Jones	income for life
3	8	2500	Comit	income for life
22	5	5000	Nikkei	income for life

These estimates of the regression coefficient values, which are equal to the constant term plus the corresponding parameters, are given in Table 9 ($\widetilde{\delta}_{kl}^{(m)}$is the coefficient relative to equation *k,* factor *m* and level *l*). The positive signs of the coefficients indicate that the respective response variables increase in relation to the level in the single product factor and vice-versa.

In order to empirically asses the predictive capacity of the estimated model, Table 10 shows the probabilities estimated through model (22) for all combinations of levels (experimental conditions) of the explanatory variables and the corresponding values of the observed proportions.

Tab. 9: Estimates of three sets of the aggregated part-worths utilities of the Coa model

Overall category	Estimated coefficient of the first equation		Overall category	Estimated coefficient of the second equation		Overall category	Estimated coefficient of the third equation	
	$\tilde{c}_1$	*-0,093*		$\tilde{c}_2$	*0,409*		$\tilde{c}_3$	*-1,955*
	$\tilde{\delta}_{12}^{(1)}$	*0,126*		$\tilde{\delta}_{22}^{(1)}$	*-0,073*		$\tilde{\delta}_{32}^{(1)}$	*-0,136*
	$\tilde{\delta}_{12}^{(2)}$	*0,058*		$\tilde{\delta}_{22}^{(2)}$	*0,181*		$\tilde{\delta}_{32}^{(2)}$	*-0,704*
	$\tilde{\delta}_{12}^{(3)}$	*-2,165*		$\tilde{\delta}_{22}^{(3)}$	*0,528*		$\tilde{\delta}_{32}^{(3)}$	*1,446*
	$\tilde{\delta}_{13}^{(3)}$	*-2,267*		$\tilde{\delta}_{23}^{(3)}$	*0,551*		$\tilde{\delta}_{33}^{(3)}$	*1,458*
	$\tilde{\delta}_{12}^{(4)}$	*0,403*		$\tilde{\delta}_{22}^{(4)}$	*-0,465*		$\tilde{\delta}_{32}^{(4)}$	*0,041*
	$\tilde{\delta}_{122}^{(12)}$	*-0,070*		$\tilde{\delta}_{222}^{(12)}$	*-0,108*		$\tilde{\delta}_{322}^{(12)}$	*0,547*
"undesira-ble"	$\tilde{\delta}_{122}^{(13)}$	*-0,120*	"desirable"	$\tilde{\delta}_{222}^{(13)}$	*0,009*	*"more desirable"*	$\tilde{\delta}_{322}^{(13)}$	*0,201*
		-0,026			*-0,122*			*0,305*
	$\tilde{\delta}_{123}^{(13)}$	*0,113*		$\tilde{\delta}_{223}^{(13)}$	*0,111*		$\tilde{\delta}_{323}^{(13)}$	*-0,640*
	$\tilde{\delta}_{122}^{(14)}$	*-0,019*		$\tilde{\delta}_{222}^{(14)}$	*0,207*		$\tilde{\delta}_{322}^{(14)}$	*0,256*
		- 0,131			*0,394*			*0,087*
	$\tilde{\delta}_{122}^{(23)}$	*0,152*		$\tilde{\delta}_{222}^{(23)}$	*-0,477*		$\tilde{\delta}_{322}^{(23)}$	*0,767*
	$\tilde{\delta}_{123}^{(23)}$	*0,935*		$\tilde{\delta}_{223}^{(23)}$	*0,747*		$\tilde{\delta}_{323}^{(23)}$	*-0,156*
	$\tilde{\delta}_{122}^{(24)}$	*1,052*		$\tilde{\delta}_{222}^{(24)}$	*0,503*		$\tilde{\delta}_{322}^{(24)}$	*-1,153*
	$\tilde{\delta}_{122}^{(34)}$			$\tilde{\delta}_{222}^{(34)}$			$\tilde{\delta}_{322}^{(34)}$	
	$\tilde{\delta}_{132}^{(34)}$			$\tilde{\delta}_{232}^{(34)}$			$\tilde{\delta}_{332}^{(34)}$	

We observe a satisfactory model fitting, as the probabilities turn out to be very near the corresponding proportions for all the level combinations of the experimental design.

Tab. 10: Comparison of the probabilities, estimated by the Coa model, and the corresponding proportions for all the level combinations of the experimental design

Experimental conditions	Proportions observed	Predicted probabilities $\pi_1(x_s)$	Proportions observed	Predicted probabilities $\pi_2(x_s)$	Proportions observed	Predicted probabilities $\pi_3(x_s)$
$z_1^{(1)}=1; z_1^{(2)}=1; z_1^{(3)}=1; z_1^{(4)}=1$	0,45	0,48	0,45	0,40	0,10	0,12
$z_1^{(1)}=1; z_2^{(2)}=1; z_2^{(3)}=1; z_1^{(4)}=1; z_{22}^{(2,3)}=1$	0,10	0,10	0,61	0,62	0,29	0,28
$z_2^{(1)}=1; z_2^{(2)}=1; z_3^{(3)}=1; z_1^{(4)}=1; z_{22}^{(1,2)}=1; z_{23}^{(1,3)}=1; z_{23}^{(2,3)}=1$	0,08	0,08	0,60	0,60	0,32	0,40
$z_2^{(1)}=1; z_2^{(2)}=1; z_2^{(3)}=1; z_1^{(4)}=1; z_{22}^{(1,2)}=1; z_{22}^{(1,3)}=1; z_{22}^{(2,3)}=1$	0,10	0,09	0,60	0,58	0,30	0,41
$z_1^{(1)}=1; z_2^{(2)}=1; z_3^{(3)}=1; z_1^{(4)}=1; z_{23}^{(2,3)}=1$	0,05	0,08	0,67	0,67	0,28	0,25
$z_2^{(1)}=1; z_1^{(2)}=1; z_1^{(3)}=1; z_1^{(4)}=1$	0,52	0,51	0,34	0,38	0,14	0,11
$z_2^{(1)}=1; z_1^{(2)}=1; z_2^{(3)}=1; z_1^{(4)}=1; z_{22}^{(1,3)}=1$	0,08	0,10	0,54	0,51	0,38	0,39
$z_2^{(1)}=1; z_1^{(2)}=1; z_3^{(3)}=1; z_1^{(4)}=1; z_{23}^{(1,3)}=1$	0,12	0,09	0,48	0,49	0,40	0,42
$z_1^{(1)}=1; z_1^{(2)}=1; z_2^{(3)}=1; z_1^{(4)}=1$	0,10	0,09	0,50	0,53	0,40	0,38
$z_1^{(1)}=1; z_2^{(2)}=1; z_1^{(3)}=1; z_1^{(4)}=1$	0,50	0,49	0,43	0,44	0,07	0,07
$z_2^{(1)}=1; z_2^{(2)}=1; z_1^{(3)}=1; z_1^{(4)}=1; z_{22}^{(1,2)}=1$	0,51	0,51	0,41	0,40	0,08	0,10
$z_1^{(1)}=1; z_1^{(2)}=1; z_3^{(3)}=1; z_1^{(4)}=1$	0,10	0,09	0,55	0,54	0,35	0,38
$z_2^{(1)}=1; z_2^{(2)}=1; z_2^{(3)}=1; z_2^{(4)}=1; z_{22}^{(1,2)}=1; z_{22}^{(1,3)}=1; z_{22}^{(1,4)}=1; z_{22}^{(2,3)}=1; z_{22}^{(2,4)}=1; z_{22}^{(3,4)}=1$	0,35	0,34	0,54	0,56	0,11	0,15
$z_2^{(1)}=1; z_1^{(2)}=1; z_3^{(3)}=1; z_2^{(4)}=1; z_{23}^{(1,3)}=1; z_{24}^{(1,4)}=1; z_{32}^{(3,4)}=1$	0,31	0,33	0,54	0,52	0,15	0,11
$z_1^{(1)}=1; z_2^{(2)}=1; z_3^{(3)}=1; z_2^{(4)}=1; z_{22}^{(2,3)}=1$	0,31	0,33	0,60	0,58	0,09	0,15
$z_1^{(1)}=1; z_1^{(2)}=1; z_1^{(3)}=1; z_2^{(4)}=1$	0,59	0,58	0,26	0,29	0,15	0,13
$z_1^{(1)}=1; z_2^{(2)}=1; z_3^{(3)}=1; z_2^{(4)}=1; z_{23}^{(2,3)}=1; z_{22}^{(2,4)}=1; z_{32}^{(3,4)}=1$	0,32	0,30	0,57	0,57	0,11	0,19
$z_1^{(1)}=1; z_1^{(2)}=1; z_2^{(3)}=1; z_2^{(4)}=1; z_{22}^{(3,4)}=1$	0,27	0,29	0,62	0,60	0,11	0,12
$z_1^{(1)}=1; z_1^{(2)}=1; z_3^{(3)}=1; z_2^{(4)}=1; z_{32}^{(3,4)}=1$	0,29	0,29	0,53	0,55	0,18	0,17
$z_2^{(1)}=1; z_2^{(2)}=1; z_1^{(3)}=1; z_2^{(4)}=1; z_{22}^{(1,2)}=1; z_{22}^{(1,4)}=1; z_{22}^{(2,4)}=1$	0,65	0,67	0,22	0,22	0,13	0,11
$z_2^{(1)}=1; z_1^{(2)}=1; z_1^{(3)}=1; z_2^{(4)}=1; z_{22}^{(1,4)}=1$	0,63	0,63	0,33	0,30	0,04	0,06
$z_2^{(1)}=1; z_2^{(2)}=1; z_3^{(3)}=1; z_2^{(4)}=1; z_{22}^{(1,2)}=1; z_{23}^{(1,3)}=1; z_{22}^{(1,4)}=1$	0,34	0,34	0,52	0,52	0,14	0,20
$z_2^{(1)}=1; z_1^{(2)}=1; z_1^{(3)}=1; z_2^{(4)}=1; z_{22}^{(1,3)}=1; z_{22}^{(1,4)}=1; z_{22}^{(3,4)}=1$	0,33	0,31	0,59	0,61	0,08	0,07
$z_1^{(1)}=1; z_2^{(2)}=1; z_1^{(3)}=1; z_2^{(4)}=1; z_{22}^{(2,4)}=1$	0,63	0,63	0,24	0,24	0,13	0,14

10. SUMMARY TABLE OF THE APPROACHES TO FULL-PROFILE CONJOINT ANALYSIS BY MULTIPLE LOGISTIC REGRESSION ANALYSIS

Table 11 shows the summary of all models characteristics of conjoint modeling by multiple logistic regression analysis so far submitted.

Tab. 11: Characteristics of conjoint modeling by multiple logistic regression analysis

Alternative approaches / *Characteristic*	1. *Ordinal logistic regression to estimate the response functions*	*2. Multivariate logistic regression to estimate the response functions*	*3. Multivariate logistic regression to estimate response functions with main and interaction effects*
1.Scaling of the response data	Multicategory assignment	Multicategory assignment	Multicategory assignment
2. Level of response aggregation	Aggregated level	Aggregated level	Aggregated level
3.Estimation method	Cumulative logit model	Constrained Non Linear Regression model	Constrained Non Linear Regression model

REFERENCES

De Luca A. (1999). Un modello di misurazione della customer satisfaction con codifica binaria additiva dei predittori ordinali, *Contributi alla Giornata di Studio*, Università di Bologna.

De Luca A. (2002). On the coincidence of the ordinary least squares estimators with the generalized least squares ones, XII Riunione Scientifica, SIS, Milano, 5-7 giugno

De Luca A. (2004). *Programmazione ed Analisi degli Esperimenti nel Marketing*, FrancoAngeli, Milano

De Luca A. and S. Ciapparelli (2006). Multivariate multiple regression on dummy variables for the estimate of the response function in the conjoint analysis, SIS, Torino

De Luca A. (2011). Ordinal logistic regression for the estimate of the response functions in the conjoint analysis, iBusiness, Scientific Research, 3, 383-389

Goldberger A.S. (1964). *Econometric Theory*, John Wiley & Sons, Inc., New York

Green P.E. and Rao V.R. (1971). Conjoint measurement for quantifying judgmental data, *Journal of Marketing Research*, 8, 355-63

Kotz S. and Johnson L. (1985). *Encyclopedia of Statistical Sciences*, Volume 5, J. Wiley & Sons, New York

Johnston J. (2000). *Econometrica*, F. Angeli, Milano